Electromagnetic Induction and Alternating Currents

This unit explores how changing magnetic environments generate electricity and the characteristics of the alternating currents (AC) that power modern civilisation.

I. Electromagnetic Induction (EMI)

Electromagnetic induction is the phenomenon where a changing magnetic field induces an electric current in a closed circuit. Experiments by Faraday and Henry demonstrated that relative motion between a magnet and a coil is a primary source of this induced current.

1. Magnetic Flux (Φ_B)

Magnetic flux through a surface of area A in a uniform magnetic field B is defined as the scalar product of the field and the area vector. $\Phi_B = \vec{B} \cdot \vec{A} = BA\cos\theta$

- **SI Unit:** Weber (Wb) or Tesla meter squared (Tm^2).
- **Nature:** Scalar quantity.

2. Faraday's Laws of Induction

- **First Law:** Whenever there is a change in the magnetic flux linked with a circuit, an electromotive force (emf) is induced.
- **Second Law:** The magnitude of the induced emf is equal to the time rate of change of magnetic flux through the circuit. $\varepsilon = -\frac{d\Phi_B}{dt}$ For a closely wound coil of N turns: $\varepsilon = -N\frac{d\Phi_B}{dt}$

3. Lenz's Law and Conservation of Energy

Lenz's law states that the polarity of an induced emf is such that it tends to produce a current which **opposes the change in magnetic flux** that produced it.

- This law is a direct consequence of the **law of conservation of energy**.
- If the induced current supported the flux change, it would create energy from nothing, violating fundamental physical laws.
- The work done by an external agent to overcome this opposition is converted into electrical energy (Joule heating).

4. Self and Mutual Induction

Inductance is the constant of proportionality between flux linkage and current, depending only on the geometry and material properties of the coil.

- **Self-Induction:** The induction of an emf in a single isolated coil due to varying current within the same coil. $N\Phi_B = LI$ $\varepsilon = -L\frac{dI}{dt}$ where L is the **self-inductance** (electrical inertia).
- **Mutual Induction:** The induction of an emf in a coil (secondary) due to a changing current in a nearby coil (primary). $N_1\Phi_1 = M_{12}I_2$ $\varepsilon_1 = -M\frac{dI_2}{dt}$ where M is the **mutual inductance**.

II. Alternating Currents (AC)

An alternating voltage is one that varies harmonically with time, usually as a sine function. $v = v_m \sin\omega t$

1. Peak and RMS Values

- **Peak Value (v_m, i_m):** The maximum amplitude of the voltage or current.
- **RMS Value (V, I):** Root mean square or "effective" value, representing the equivalent DC value that produces the

same average power dissipation. $I = \frac{i_m}{\sqrt{2}} \approx 0.707 i_m$ $V = \frac{v_m}{\sqrt{2}} \approx 0.707 v_m$

2. Phasors

Phasors are vectors that rotate about the origin with angular speed ω. Their vertical projections represent the instantaneous values of sinusoidally varying quantities like voltage and current.

3. Reactance and Impedance

Different circuit elements respond differently to AC:

- **Pure Resistor:** Voltage and current are **in phase** ($\phi = 0$).
- **Pure Inductor:** Current **lags** voltage by $\pi/2$. Inductive Reactance is: $X_L = \omega L$
- **Pure Capacitor:** Current **leads** voltage by $\pi/2$. Capacitive Reactance is: $X_C = \frac{1}{\omega C}$
- **LCR Series Circuit:** The total opposition to current is **Impedance** (Z). $Z = \sqrt{R^2 + (X_C - X_L)^2}$ The phase difference (ϕ) between voltage and current is: $\tan\phi = \frac{X_C - X_L}{R}$

4. Resonance

Resonance occurs in a series LCR circuit when inductive and capacitive reactances cancel each other ($X_L = X_C$).

- **Condition:** $\omega_0 L = \frac{1}{\omega_0 C}$.
- **Resonant Frequency:** $\omega_0 = \frac{1}{\sqrt{LC}}$.
- At resonance, impedance is minimum ($Z = R$), and current amplitude is maximum.

5. Power in AC Circuits

The average power dissipated in an AC circuit depends on the phase angle. $P = VI\cos\phi$

- **Power Factor:** The term $\cos\phi$. For a purely resistive circuit, $\cos\phi = 1$ (maximum power). For purely inductive/capacitive circuits, $\cos\phi = 0$.
- **Wattless Current:** Current in a circuit where no power is dissipated ($\phi = \pi/2$), typically in pure inductors or capacitors.

III. Electrical Devices

1. AC Generator

An **AC generator** converts mechanical energy into electrical energy using the principle of electromagnetic induction.

- As a coil (armature) of N turns and area A rotates with angular speed ω in a uniform magnetic field B, the changing flux induces an alternating emf.
- **Instantaneous emf:** $\varepsilon = NBA\omega \sin\omega t$.
- The frequency of rotation is 50 Hz in India and 60 Hz in the USA.

2. Transformer

A **transformer** changes alternating voltage levels using the principle of mutual induction. It consists of a primary coil (N_p) and a secondary coil (N_s) wound on a soft-iron core.

- **Voltage Ratio:** $\frac{V_s}{V_p} = \frac{N_s}{N_p}$
- **Current Ratio:** For an ideal (100% efficient) transformer, $I_pV_p = I_sV_s$, so: $\frac{I_p}{I_s} = \frac{N_s}{N_p}$
- **Step-up Transformer:** $N_s > N_p$; increases voltage and decreases current.
- **Step-down Transformer:** $N_s < N_p$; decreases voltage and increases current.
- **Energy Losses:** Real transformers have small losses due to flux leakage, resistance of windings, eddy currents, and hysteresis.
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