

1. Introduction and Related Terminology

To understand Linear Programming (LP), students must be familiar with its foundational terminology:

- **Objective Function:** This is the linear function $Z = ax + by$, where a and b are constants, which must be maximised or minimised. In a business context, this usually represents profit or cost.
- **Decision Variables:** The variables x and y in the objective function are called decision variables. Their values are what a programmer needs to determine to achieve the optimal result.
- **Constraints:** These are the linear inequalities or equations that restrict the values of the decision variables. For instance, limitations on resources like capital, labour, or storage space are expressed as constraints.
- **Non-negative Restrictions:** In most real-life scenarios, variables cannot be negative (e.g., you cannot produce a negative number of chairs). These are written as: $x \geq 0$, $y \geq 0$.
- **Optimisation Problem:** A general class of problems that seeks the "best" (optimal) value of a function within a given set of conditions. Linear programming problems are a specific subset where the objective function and constraints are all linear.
- **Feasible Region:** The common region determined by all constraints, including non-negative restrictions, is the feasible region. Every point in this region is a **feasible solution**.
- **Infeasible Region:** Any region outside the feasible region is called the infeasible region, and points within it are **infeasible solutions** that do not satisfy at least one constraint.
- **Optimal Feasible Solution:** This is a point in the feasible region that produces the maximum (or minimum) value of the objective function.

2. Mathematical Formulation of LPP

Formulating a Linear Programming Problem (LPP) involves translating a practical situation into a mathematical model. The sources illustrate this using the example of a furniture dealer who has $Rs\ 50,000$ to invest and storage space for at most 60 pieces.

Example Formulation:

1. **Identify Decision Variables:** Let x be the number of tables and y be the number of chairs.
2. **State Non-negative Constraints:** Since the dealer cannot buy negative items, $x \geq 0$ and $y \geq 0$.
3. **Establish Linear Constraints:**
 - **Investment constraint:** If a table costs $Rs\ 2500$ and a chair costs $Rs\ 500$, the total cost must not exceed the capital:
 $2500x + 500y \leq 50000 \implies 5x + y \leq 100$
 - **Storage constraint:** The total number of items must not exceed 60:
 $x + y \leq 60$.
4. **Define the Objective Function:** If the profit is $Rs\ 250$ per table and $Rs\ 75$ per chair, the total profit Z is: Maximise $Z = 250x + 75y$.

3. Graphical Method of Solution

For problems involving only **two variables**, the graphical method is the primary tool for finding a solution. This method relies on graphing the system of linear inequalities on a coordinate plane to identify the feasible region.

Fundamental Theorems

The graphical method is supported by two key theorems:

- **Theorem 1:** If a linear programming problem has an optimal value (maximum or minimum), this value **must occur at a corner point (vertex)** of the feasible region.

- **Theorem 2:** If the feasible region is **bounded**, the objective function always has both a maximum and a minimum value, both occurring at corner points.

The Corner Point Method

This is the standard procedure for solving an LPP graphically:

1. **Graph the System:** Draw lines for each constraint and identify the common shaded region (the feasible region).
2. **Find Corner Points:** Determine the coordinates of all vertices of the feasible region. This can be done via inspection or by solving the simultaneous equations of intersecting lines.
3. **Evaluate the Objective Function:** Substitute the coordinates of each corner point into $Z = ax + by$.
4. **Identify Optimal Values:** The largest value obtained is the maximum, and the smallest is the minimum.

Feasible Region (Convexity)

The feasible region for any LPP is always a **convex region**. This means if you take any two points in the region, the line segment joining them lies entirely within the region.

- **Bounded Regions:** A feasible region is bounded if it can be enclosed within a circle. In these cases, a maximum and minimum value are guaranteed to exist at corner points.
- **Unbounded Regions:** An unbounded region extends indefinitely in at least one direction. In such cases, an optimal value might not exist.

Infeasible Regions

If there is no point that satisfies all constraints simultaneously, the problem has **no feasible region**. Consequently, there is no feasible solution and no optimal value. This often occurs when constraints are contradictory, such as requiring $x + y \geq 8$ while also requiring $3x + 5y \leq 15$ for positive variables.

5. Optimal Feasible Solutions (Bounded vs. Unbounded)

Finding the optimal feasible solution requires careful handling of the region's boundaries.

Case 1: Bounded Region

In a bounded region, the process is straightforward: the maximum and minimum are the highest and lowest values found at the vertices.

Example (from source): For Maximise $Z = 4x + y$ with constraints $x + y \leq 50$ and $3x + y \leq 90$:

- Corner points: $(0, 0)$, $(30, 0)$, $(20, 30)$, $(0, 50)$.
- Values of Z : $0, 120, 110, 50$.
- **Maximum value is 120 at $(30, 0)$.**

Case 2: Unbounded Region



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4. Feasible and Infeasible Regions

If the region is unbounded, a corner point value M is the maximum **only if** the open half-plane determined by $ax + by > M$ has **no points in common** with the feasible region. Similarly, for a minimum value m , the half-plane $ax + by < m$ must have no common points with the feasible region. If there are common points, the optimal value does not exist.

Case 3: Multiple Optimal Solutions

If two corner points produce the same maximum (or minimum) value, then **every point on the line segment** joining those two vertices is also an optimal solution. For example, if both points $(15, 15)$ and $(0, 20)$ give a value of 180, all points on the segment between them are optimal.

6. LPP with Three Non-Trivial Constraints

The CUET syllabus requires students to handle up to three non-trivial constraints. The complexity arises in identifying the vertices formed by the intersection of these lines.

Example Analysis (based on source logic):

Consider:

1. $x + 3y \leq 60$
2. $x + y \geq 10$
3. $x \leq y$ (Plus non-negative constraints $x, y \geq 0$).

In this scenario, the feasible region is formed by the intersection of three lines. The vertices are found by solving pairs of equations:

- Intersection of (2) and (3) gives corner point $(5, 5)$.
- Intersection of (1) and (3) gives corner point $(15, 15)$.
- The vertices on the y-axis are $(0, 10)$ and $(0, 20)$.

By evaluating the objective function at all four points, one can determine the specific point of optimisation.

7. Summary Table for CUET Preparation

Term	Mathematical Definition / Logic
Linear Objective Function	$Z = ax + by$
Feasible Region	Common area satisfying all linear constraints
Convexity	Property of feasible regions where the region is "bulged out"
Corner Point Method	Testing vertices to find the optimal Z
Bounded Region	Region that can be enclosed; always has max and min
Unbounded Region	Region extending indefinitely; max/min may not exist
Infeasibility	Occurs when no common region satisfies all constraints
Multiple Solutions	Occurs when two vertices have the same optimal value