

## The Production Function

**Production** is the process by which a firm transforms **inputs** (factors of production) into **outputs** that can be consumed or used for further production.

A **production function** specifically describes the relationship between the quantities of inputs used and the maximum quantity of output that can be produced from those inputs given a specific technology.

For a firm using two factors, **Labour (L)** and **Capital(K)**, the function is expressed as:

$$q=f(L,K)$$

Where  $q$  is the maximum output. If technology improves, a new production function is created because the same level of inputs can produce more output.

### Short Run vs. Long Run Production Function

- **Short Run:** A time period where at least one factor of production (usually capital) remains fixed and cannot be varied. To change output, the firm can only vary the **variable factor** (usually labour).
- **Long Run:** A time period in which **all factors** of production can be varied. There are no fixed factors in the long run.

### Returns to Scale (Long Run Only)

In the long run, when all inputs are increased by the same proportion ( $t$ ), the resulting output change determines the **Returns to Scale**:

1. **Constant Returns to Scale (CRS):** Output increases by the same proportion as inputs ( $f(tL,tK)=t \cdot f(L,K)$ ).
2. **Increasing Returns to Scale (IRS):** Output increases by a larger proportion than inputs ( $f(tL,tK)>t \cdot f(L,K)$ ).
3. **Decreasing Returns to Scale (DRS):** Output increases by a smaller proportion than inputs ( $f(tL,tK)<t \cdot f(L,K)$ ).

## 2. Total, Marginal, and Average Product

When studying the short-run production function (where capital is fixed), three primary concepts are used to measure the contribution of the variable input:

- **Total Product (TP):** The relationship between the variable input and total output, keeping all other inputs constant. TP is the sum of all Marginal Products.
- **Average Product (AP):** Output per unit of the variable input ( $APL=TP/L$ ).
- **Marginal Product (MP):** The change in output resulting from a unit change in the variable input when all other inputs are held constant ( $MPL=\Delta TP/\Delta L$ ).

### 3. Shapes of TP, MP, and AP Curves

The shapes of these curves are governed by the **Law of Variable Proportions** (also known as the Law of Diminishing Marginal Product).

#### The Law of Variable Proportions

This law states that as we increase the employment level of a variable factor (holding others fixed), the MP initially rises but eventually starts falling. This occurs because:

1. Initially, factor proportions become more suitable for production.
2. Eventually, the production process becomes "too crowded" with the variable input, leading to inefficiency.

#### Curve Characteristics

- **TP Curve:** A positively sloped curve that increases as the variable input increases, though the rate of increase varies based on the MP.
- **MP Curve:** It is **inverse 'U'-shaped**. It rises initially, reaches a maximum, and then falls as diminishing returns set in.
- **AP Curve:** Also **inverse 'U'-shaped**. It represents the average of all marginal products up to a certain level.

#### Relationships between the Curves

- MP and AP start at the same point for the first unit of input.
- As long as **MP > AP**, the **AP rises**.
- When **MP < AP**, the **AP falls**.

- **Crucial Point:** The MP curve cuts the AP curve from above at the **maximum point of the AP curve**.



#### 4. Concepts of Cost

To produce output, firms must pay for inputs, which constitutes the **cost of production**. A **cost function** describes the least cost of producing each level of output given input prices and technology.

#### Short-Run Cost Concepts

1. **Total Fixed Cost (TFC):** Costs incurred to employ fixed inputs. These costs do not change with the level of output and remain constant even at zero output. Graphically, TFC is a **horizontal straight line**.
2. **Total Variable Cost (TVC):** Costs incurred to employ variable inputs. TVC increases as output increases.
- 3.
4. **Total Cost (TC):** The sum of fixed and variable costs ( $TC=TFC+TVC$ ). TC increases as output increases.
5. **Average Fixed Cost (AFC):** TFC per unit of output ( $AFC=TFC/q$ ). Since TFC is constant, AFC decreases as output increases, forming a **rectangular hyperbola**.

6. **Average Variable Cost (AVC):** TVC per unit of output ( $AVC=TVC/q$ ).
7. **Short-Run Average Cost (SAC):** Total cost per unit of output ( $SAC=TC/q$  or  $SAC=AFC+AVC$ ).
8. **Short-Run Marginal Cost (SMC):** The change in total cost per unit change in output ( $SMC=\Delta TC/\Delta q$ ). In the short run, SMC is entirely due to changes in TVC.

#### 5. Shapes of Short-Run Cost Curves

Short-run cost curves (except AFC) are generally **'U'-shaped** due to the Law of Variable Proportions.

- **SMC Curve:** Initially falls and then rises as diminishing marginal product sets in.
- **AVC Curve:** Initially falls, reaches a minimum, and then rises as SMC becomes greater than AVC.
- **SAC Curve:** It is the vertical sum of AVC and AFC. It also falls initially, reaches a minimum point (to the right of AVC's minimum), and then rises.
- **Relationship between Marginal and Average Costs:**
  - SMC cuts the AVC curve from below at the **minimum point of AVC**.
  - SMC cuts the SAC curve from below at the **minimum point of SAC**.
  - When SMC is less than Average Cost (SAC/AVC), the Average Cost is falling; when SMC is greater, the Average Cost is rising.

#### 6. Concepts of Revenue

**Revenue** is the money earned by a firm through selling its output in the market.

- **Total Revenue (TR):** The market price ( $p$ ) multiplied by the quantity sold ( $q$ ) ( $TR=p \times q$ ). Under perfect competition, price is constant, so TR is an **upward-rising straight line** starting from the origin.
- **Average Revenue (AR):** Total revenue per unit of output ( $AR=TR/q$ ). For a price-taking firm, **AR**

= **Market Price ( $p$ )**. The AR curve is a horizontal line at the market price, often called the **price line**.

- **Marginal Revenue (MR)**: The increase in total revenue for a unit increase in output ( $MR = \Delta TR / \Delta q$ ). For a perfectly competitive firm, **MR =  $p$** , and thus **AR = MR =  $p$** .



### 3. Condition 3 (Continuity of Production):

- **Short Run**: The market price must be greater than or equal to the minimum Average Variable Cost ( $P \geq AVC$ ). If  $P < AVC$ , the firm's total revenue cannot even cover its variable costs, and it will minimize losses by shutting down (producing zero output).
- **Long Run**: The market price must be greater than or equal to the minimum Average Cost ( $P \geq AC$ ). If  $P < AC$ , the firm incurs a loss and will exit the market in the long run.

### Key Equilibrium Points

- **Shut-down Point**: In the short run, this is the point of minimum AVC.
- **Break-even Point**: The point where the firm earns only **normal profit** (where  $P = \text{minimum SAC or } LAC$ ). **Normal profit** is the minimum level needed to keep the firm in business and is considered part of total costs. Profits earned above this are called **super-normal profits**.

## 7. Producer's Equilibrium: Profit Maximisation

A producer is in equilibrium when they produce the quantity that maximizes **Profit ( $\pi$ )**, which is defined as  $TR - TC$ .

### The MC and MR Approach

For a firm to maximize profit, three conditions must be satisfied simultaneously at the chosen output level ( $q_0$ ):

1. **Condition 1 ( $MR = MC$ )**: Profit is increasing as long as  $MR > MC$  and falling if  $MR < MC$ . Therefore, at equilibrium,  $MR$  must equal  $MC$ . Since  $MR = P$  under perfect competition, this implies  $P = MC$ .
2. **Condition 2 ( $MC$  must be non-decreasing)**: The  $MC$  curve must be **upward-sloping** at the equilibrium point. A firm will not produce in the range where  $MC$  is falling because increasing output would further reduce costs and increase profits.