1. Conditional Probability

In many scenarios, the occurrence of one event affects the likelihood of another. **Conditional probability** is the probability of an event E occurring, given that another event F has already occurred. E occurring, given that another event F has already occurred.

A. Intuitive Understanding: The Reduced Sample Space

When we know event F has occurred, any outcomes in the original sample space S that are not in F are no longer possible. This information effectively **reduces the sample space** from S to the subset F. The only outcomes of E that remain relevant are those that are also in F, which is the intersection $E \cap F$. F has occurred, any outcomes in the original sample space S that are not in F are no longer possible. This information effectively reduces the sample space from S to the subset F. The only outcomes of E that remain relevant are those that are also in F, which is the intersection $E \cap F$.

B. Formal Definition

For two events E and F associated with the same sample space, where $P(F) \neq 0$, the conditional probability is defined as: E and F associated with the same sample space, where $P(F) \neq 0$, the conditional probability is defined as: $P(E|F) = \frac{P(E \cap F)}{P(F)}$

C. Properties of Conditional Probability

1. **Property 1:** The probability of the sample space S given an event F is 1, and the probability of F given F is also 1. S given an event F is 1, and the probability of F given F is also 1. $P(S|F) = P(F|F) = 1$
2. **Property 2:** If A and B are any two events and F is a conditioning event, the probability of their union is: A and B are any two events and F is a conditioning event, the probability of their union is: $P((A \cup B)|F) = P(A|F) + P(B|F) - P((A \cap$
   Note: If A and B are **disjoint (mutually exclusive)**, then $P((A \cap B)|F) = 0$. A and B are disjoint (mutually exclusive), then $P((A \cap B)|F) = 0$.
3. **Property 3:** The probability of the complement of an event E given F is: E given F is: $P(E'|F) = 1 - P(E|F)$

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2. Multiplication Theorem on Probability

The **multiplication theorem** allows us to find the probability of the **simultaneous occurrence** of two or more events, denoted by $E \cap F$ or simply EF. $E \cap F$ or simply EF.

A. Derivation for Two Events

From the definition of conditional probability, we can derive the multiplication rule: $P(E \cap F) = P(E) \cdot P(F|E), \quad P(E) \neq 0$
Alternatively: $P(E \cap F) = P(F) \cdot P(E|F), \quad P(F) \neq 0$

B. Extension to Multiple Events

The rule can be extended to three or more events by applying it sequentially: $P(E \cap F \cap G) = P(E) \cdot P(F|E) \cdot P(G|E \cap F)$ In this notation, $P(G|E \cap F)$ is the probability that G occurs given that both E and F have already occurred. $P(G|E \cap F)$ is the probability that G occurs given that both E and F have already occurred.

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3. Independent Events

Two events are **independent** if the occurrence of one does not affect the probability of the occurrence of the other.

A. Formal Definition

Events E and F are independent if: E and F are independent if:

1. $P(E|F) = P(E)$ $P(E|F) = P(E)$
2. $P(F|E) = P(F)$ $P(F|E) = P(F)$

Using the multiplication theorem, this leads to the **product rule for independence**: $P(E \cap F) = P(E) \cdot P(F)$

B. Independent vs. Mutually Exclusive

It is a common mistake to confuse these terms.

- **Mutually Exclusive:** Events cannot happen at the same time (no outcomes in common).
- **Independent:** The probability of one does not change the probability of the other.
- **Crucial Difference:** Two independent events with non-zero probabilities **cannot** be mutually exclusive, as they must be able to occur together to satisfy the product rule.

C. Mutual Independence of Three Events

Three events $A, B, \text{ and } C$ are **mutually independent** if they satisfy four conditions: $A, B, \text{ and } C$ are mutually independent if they satisfy four conditions:

1. $P(A \cap B) = P(A)P(B)$
   $P(A \cap B) = P(A)P(B)$
2. $P(A \cap C) = P(A)P(C)$
   $P(A \cap C) = P(A)P(C)$
3. $P(B \cap C) = P(B)P(C)$
   $P(B \cap C) = P(B)P(C)$
4. $P(A \cap B \cap C) = P(A)P(B)P(C)$ If even one of these conditions fails, the events are not mutually independent. $P(A \cap B \cap C) = P(A)P(B)P(C)$ If even one of these conditions fails, the events are not mutually independent.

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4. Total Probability

Before calculating "reverse" probabilities, we must understand how to calculate the total probability of an event that can happen under different scenarios.

A. Partition of a Sample Space

A set of events $E_1, E_2, \dots, E_n$ is a **partition** of sample space S if: $E_1, E_2, \dots, E_n$ is a partition of sample space S if:

- They are pairwise disjoint ($E_i \cap E_j = \phi$). $E_i \cap E_j = \phi$).
- They are exhaustive ($E_1 \cup E_2 \cup \dots \cup E_n = S$). $E_1 \cup E_2 \cup \dots \cup E_n = S$).
- Each has a non-zero probability ($P(E_i) > 0$). $P(E_i) > 0$).

B. Theorem of Total Probability

If $E_1, E_2, \dots, E_n$ is a partition of S, then for any event A associated with S, the total probability $P(A)$ is the sum of the probabilities of A occurring under each partition. $E_1, E_2, \dots, E_n$ is a partition of S, then for any event A associated with S, the total probability $P(A)$ is the sum of the probabilities of A occurring under each partition.

$P(A) = \sum_{j=1}^{n} P(E_j) \cdot P(A|E_j)$ This formula works because A can be decomposed into disjoint components $(A \cap E_1), (A \cap E_2), \ldots, (A \cap E_n)$. A can be decomposed into disjoint components $(A \cap E_1), (A \cap E_2), \ldots, (A \cap E_n)$.

5. Bayes' Theorem

Named after **John Bayes**, this theorem provides a way to find the **reverse probability**: the probability of a specific "cause" (partition event E_i) given that the "effect" (event A) has already occurred. E_i) given that the "effect" (event A) has already occurred.

A. The Formula

Let $E_1, E_2, \ldots, E_n$ be a partition of S, and A be an event with non-zero probability. The probability of E_i given A is: $E_1, E_2, \ldots, E_n$ be a partition of S, and A be an event with non-zero probability. The probability of E_i given A is:

$$P(E_i|A) = \frac{P(E_i) \cdot P(A|E_i)}{\sum_{j=1}^{n} P(E_j) \cdot P(A|E_j)}$$

B. Terminology

- **Hypotheses:** The events $E_1, E_2, \ldots, E_n$ represent the possible causes. $E_1, E_2, \ldots, E_n$ represent the possible causes.
- **Priori Probability:** The initial probability $P(E_i)$ determined before the experiment. $P(E_i)$ determined before the experiment.
- **Posteriori Probability:** The updated conditional probability $P(E_i|A)$ calculated after observing event A. $P(E_i|A)$ calculated after observing event A.

6. Summary Table for CUET Preparation

Concept	Key Formula	Context / Usage
Conditional Prob.	$P(E$	F) = \frac{P(E \cap F)}{P(F)} $F)=\frac{P(E\cap F)}{P(F)}$$$ = \frac{P(E \cap F)}{P(F)} $=\frac{P(E\cap F)}{P(F)}$$$ \frac{P(E \cap F)}{P(F)} $\frac{P(E\cap F)}{P(F)}$$$ frac{P(E \cap F)}{P(F)} $fracP(E\cap F)P(F)$ ac{P(E \cap F)}{P(F)} $acP(E\cap F)P(F)$$$ {P(E \cap F)}{P(F)} $P(E\cap F)P(F)$$$ (E \cap F)}{P(F)} \cap F)}{P(F)} cap F)}{P(F)} p F)}{P(F)} F)}{P(F)} }{P(F)} P(F)} F)} } $$
Multiplication Rule	$$P(E \cap F) = P(E)P(F	E)$ $$
Independence	$P(E \cap F) = P($	Events do not influence each other.
Total Probability	mimeTeX failed to render your expression P(E_j)P(A	E_j)$j)$ $$
Bayes' Theorem	$\frac{P(E}{[?]}$)P(A	E_i)}{P(A)}$$$$

7. Numerical Application Insights (Based on Sources)

- **Tree Diagrams:** Useful for visualizing experiments like tossing a coin and then throwing a die depending on the coin outcome.
- **With/Without Replacement:** In card or ball drawing problems, "without replacement" implies that the events are **dependent**, requiring conditional probability for the second draw.
- **Truth-telling Problems:** Bayes' theorem is often applied to find the probability that a report is actually true (e.g., a man reports a six occurred; what is the probability it actually was a six?).
- **Reliability Tests:** Standard in Bayes' problems involving medical tests (HIV/disease detection) where false positives and false negatives are given.