Electrostatics: Comprehensive Study Notes

Electrostatics is the branch of physics dealing with the forces, fields, and potentials arising from **static charges**—charges that do not change or move with time.

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1. Electric Charges and Basic Properties

Electric charge is an intrinsic property of matter. There are two types of charges: **positive** and **negative**.

- **Fundamental Law:** Like charges repel each other, while unlike charges attract each other.
- **Additivity of Charges:** Total charge in a system is the algebraic sum of individual point charges. For n charges, the total charge q is: $q = q_1 + q_2 + q_3 + \cdots + q_n$
- **Conservation of Charge:** In an isolated system, the total charge remains constant. Charges can be transferred between bodies but neither created nor destroyed.
- **Quantisation of Charge:** All free charges are integral multiples of a basic unit of charge, e $(1.602 \times 10^{-19}\ \mathrm{C})$. $q = ne$ where $n = 0, \pm 1, \pm 2, \ldots$.

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2. Coulomb's Law and Superposition

Coulomb's Law describes the electrostatic force between two point charges at rest.

- **Magnitude:** The force F between charges q_1 and q_2 separated by distance r in vacuum is: $F = \frac{1}{4\pi\epsilon_0}\frac{q_1 q_2}{r^2}$ where $\epsilon_0 \approx 8.854 \times 10^{-12}\ \mathrm{C}^2\ \mathrm{N}^{-1}\ \mathrm{m}^{-2}$ is the **permittivity of free space**.
- **Vector Form:** The force $\vec{F}21$ *on* q_2 *due to* q_1 *is:* $\vec{F}21 = \frac{1}{4\pi\epsilon_0}\frac{q_1 q_2}{r_{21}^2}\hat{r}_{21}$.
- **Principle of Superposition:** The total force on a charge due to multiple other charges is the vector sum of individual Coulomb forces exerted by each. $\vec{F}1 = \vec{F}12 + \vec{F}13 + \cdots + \vec{F}1n$.

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3. Continuous Charge Distributions

For large-scale systems, we treat charge as continuous rather than discrete.

- **Linear Charge Density (λ):** $\lambda = \frac{\Delta Q}{\Delta l}$ (C/m).
- **Surface Charge Density (σ):** $\sigma = \frac{\Delta Q}{\Delta S}$ (C/m^2).
- **Volume Charge Density (ρ):** $\rho = \frac{\Delta Q}{\Delta V}$ (C/m^3).

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4. Electric Field and Field Lines

The **Electric Field ($\vec{E}$)** at a point is the force per unit positive test charge placed at that point.

- **Due to Point Charge:** $\vec{E} = \frac{1}{4\pi\epsilon_0}\frac{Q}{r^2}\hat{r}$.
- **Field Lines:** Pictorial representations where the tangent shows field direction and density shows field strength.
  - Lines originate from positive and terminate at negative charges.
  - Lines never cross and do not form closed loops.

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5. Electric Dipole

An **electric dipole** is a pair of equal and opposite charges ($q, -q$) separated by a small distance $2a$.

- **Dipole Moment ($\vec{p}$):** A vector directed from $-q$ to $+q$. $\vec{p} = q(2\vec{a})$.
- **Field of a Dipole (for $r \gg a$):**
  - **Axial Point:** $\vec{E} \approx \frac{1}{4\pi\epsilon_0}\frac{2\vec{p}}{r^3}$.
  - **Equatorial Point:** $\vec{E} \approx -\frac{1}{4\pi\epsilon_0}\frac{\vec{p}}{r^3}$.
- **Torque in Uniform Field:** $\vec{\tau} = \vec{p} \times \vec{E}$. Net force is zero.

6. Electric Flux and Gauss's Theorem

Electric Flux (Φ) measures the number of field lines crossing a surface area. $\Delta\Phi = \vec{E}\cdot\Delta\vec{S} = E\Delta S\cos\theta$.

- **Gauss's Theorem:** The total flux through any closed surface is $1/\epsilon_0$ times the total charge q enclosed. $\Phi = \oint \vec{E}\cdot d\vec{A} = \frac{q}{\epsilon_0}$.

Applications:

1. **Infinite Long Wire:** $E = \frac{\lambda}{2\pi\epsilon_0 r}$.
2. **Infinite Plane Sheet:** $E = \frac{\sigma}{2\epsilon_0}$.
3. **Thin Spherical Shell (Radius R):**
   - **Outside ($r \geq R$):** $E = \frac{1}{4\pi\epsilon_0}\frac{q}{r^2}$.
   - **Inside ($r < R$):** $E = 0$.

7. Electrostatic Potential and Energy

Electrostatic Potential (V) is the work done per unit positive charge in bringing it from infinity to a point.

- **Potential Difference ($V_P - V_R$):** $V_P - V_R = \frac{U_P - U_R}{q}$.
- **Due to Point Charge:** $V = \frac{1}{4\pi\epsilon_0}\frac{Q}{r}$.
- **Due to Dipole:** $V \approx \frac{1}{4\pi\epsilon_0}\frac{\vec{p}\cdot\hat{r}}{r^2}$.
- **Equipotential Surfaces:** Surfaces where potential is constant; the electric field is always normal to them.
- **Relation between E and V:** $E = -\frac{\delta V}{\delta l}$.
- **Potential Energy (U):**
  - **Two charges:** $U = \frac{1}{4\pi\epsilon_0}\frac{q_1 q_2}{r_{12}}$.
  - **Dipole in Field:** $U = -\vec{p}\cdot\vec{E}$.

8. Conductors and Dielectrics

- **Conductors:** Contain free electrons. Field inside is zero; charges reside on surface; potential is constant throughout.
- **Electrostatic Shielding:** Field inside a cavity of a conductor is always zero.
- **Dielectrics:** Non-conducting substances where external fields induce dipole moments (**polarization**).
- **Polarisation ($\vec{P}$):** Dipole moment per unit volume.

9. Capacitors and Capacitance

A **capacitor** stores charge and electrical energy. $C = \frac{Q}{V}$. Unit: **Farad (F)**.

- **Parallel Plate Capacitor:** For plates of area A and separation d:
  - **Vacuum:** $C_0 = \frac{\epsilon_0 A}{d}$.
  - **With Dielectric (K):** $C = \frac{K\epsilon_0 A}{d} = KC_0$.
- **Combinations:**
  - **Series:** $\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} + \dots$.
  - **Parallel:** $C = C_1 + C_2 + \dots$.
- **Energy Stored (U):** $U = \frac{1}{2}QV = \frac{1}{2}CV^2 = \frac{Q^2}{2C}$.
- **Energy Density (u):** Energy per unit volume is $u = \frac{1}{2}\epsilon_0 E^2$.