Short Notes of Mathematics for CUET(B2)

UNIT VII: FINANCIAL MATHEMATICS

A. Perpetuity and Sinking Funds

1. Concept of Perpetuity

A **perpetuity** is a type of annuity that receives or pays a fixed amount of money periodically and continues **indefinitely** (forever). Because it has no end date, we cannot calculate its "future value." Instead, we calculate its **Present Value (PV)**.

Note from outside the sources: The Present Value of a perpetuity is the sum of an infinite geometric series. The sources discuss infinite G.P. sums, noting that if $|r| < 1$, the sum is $S = \frac{a}{1-r}$. In financial terms, this translates to: $PV = \frac{R}{i}$ Where:

- R = Periodic payment
- i = Interest rate per period

2. Sinking Funds

A **sinking fund** is a fund established by an organization or government for retiring debt or replacing a wasting asset. It involves setting aside a fixed amount of money periodically, which earns compound interest, to reach a target amount in the future.

3. Difference: Sinking Fund vs. Saving Account

- **Purpose:** A sinking fund is created for a specific, predetermined future liability (e.g., replacing machinery or paying a bond). A savings account is general-purpose.
- **Obligation:** Sinking funds are often a legal requirement in bond contracts to protect lenders.
- **Calculation:** Sinking fund payments are calculated using the "Future Value of an Annuity" formula to ensure the target is met exactly.

B. Calculation of EMI (Equated Monthly Installment)

1. Concept of EMI

An **EMI** is a fixed payment amount made by a borrower to a lender at a specified date each calendar month. EMIs are used to pay off both **interest and principal** each month, so that over a specified number of years, the loan is paid off in full.

2. Methods of Calculation

Note from outside the sources: There are two primary methods for calculating interest, which affect the EMI.

- **Flat Rate Method:** Interest is calculated on the full original loan amount throughout the tenure.
- **Reducing Balance Method:** Interest is calculated only on the remaining outstanding principal. This is the standard method for most modern loans.

EMI Formula (Reducing Balance): $E = P \cdot r \cdot \frac{(1+r)^n}{(1+r)^n - 1}$ Where:

- P = Principal loan amount
- r = Monthly interest rate (Annual rate / 12)
- n = Loan tenure in months

C. Calculation of Returns and Nominal Rate of Return

Returns measure the efficiency or profitability of an investment.

1. Rate of Return (RoR)

RoR is the net gain or loss of an investment over a specified time period, expressed as a percentage of the investment's initial cost. **Note from outside the sources:**

$$RoR = \frac{\text{Current Value} - \text{Initial Value}}{\text{Initial Value}} \cdot 100$$

2. Nominal Rate of Return

The **Nominal Rate** is the "stated" interest rate of an investment without adjusting for inflation or compounding within the year. The sources demonstrate how a principal P can increase continuously at a rate $r\%$, modeled by the differential equation $\frac{dP}{dt} = \frac{rP}{100}$. Solving this shows that the growth is exponential: $P = C \cdot e^{\frac{rt}{100}}$ where r is the nominal rate.

D. Compound Annual Growth Rate (CAGR)

1. Concept of CAGR

CAGR represents the mean annual growth rate of an investment over a specified period longer than one year. It "smooths" out the returns, assuming the investment grew at a steady rate on a compounded basis.

2. Difference between CAGR and AGR

- **Annual Growth Rate (AGR):** Measures the growth over a single year. It can be highly volatile.
- **CAGR:** Describes the rate at which an investment would have grown if it had grown at a steady rate each year.

3. Calculation of CAGR

Note from outside the sources:

$$CAGR = \left[\left(\frac{EV}{BV} \right)^{\frac{1}{n}} - 1 \right] \cdot 100$$

Where:

- EV = Ending Value
- BV = Beginning Value
- n = Number of years

E. Linear Method of Depreciation

Depreciation is the systematic reduction in the recorded cost of a fixed asset.

1. Concept of Linear (Straight-Line) Depreciation

In the linear method, the asset loses an **equal amount of value** every year. This relates to the mathematical concept of a **constant rate of change** (slope) discussed in the application of derivatives.

2. Key Terms

- **Cost (C):** The original purchase price of the asset.
- **Residual Value (S):** The estimated value of the asset at the end of its useful life (also called Scrap Value).
- **Useful Life (n):** The period over which the asset is expected to be used.

3. Calculation of Annual Depreciation

If D is the annual depreciation: $D = \frac{C - S}{n}$ The book value (V) at any time t is a linear function: $V(t) = C - Dt$. This matches the linear model $y = mx + c$ where the slope $m = -D$.

F. Valuation of Bonds

1. Concept and Terms

A **bond** is a fixed-income instrument that represents a loan made by an investor to a borrower.

- **Face Value (Par Value):** The amount the bond will be worth at maturity.
- **Coupon Rate:** The interest rate the bond issuer pays to the bondholder.
- **Maturity Date:** The date when the principal is paid back.

2. Present Value Approach to Bond Valuation

The value of a bond is the **Present Value** of all its future cash flows (coupon payments and the final face value).

Note from outside the sources: This involves "discounting" future values back to the present using a required rate of return (r). You may want to independently verify the Bond Pricing formula:

$$Value = \sum_{t=1}^{n} \frac{C}{(1+r)^t} + \frac{F}{(1+r)^n}$$

Where:

- C = Periodic coupon payment
- r = Required yield/discount rate
- F = Face value of the bond
- n = Number of periods to maturity

This approach is a direct application of the **Fundamental Theorem of Calculus** principles—summing up infinitesimal values (or in this case, discrete time periods) to find a total current value.

Summary Table: Financial Math Formulae for CUET

Topic	Formula	Mathematical Context
Perpetuity PV	$PV = \frac{R}{i}$	Infinite G.P. Sum
Loan Growth	$P = P_0 e^{rt}$	Exponential Function
Depreciation	$D = \frac{C - S}{n}$	Constant Rate of Change
EMI	$E = P \cdot r \cdot \frac{(1+r)^n}{(1+r)^n - 1}$	Finite Annuity Sum
Bond Value	$\sum \frac{CF_t}{(1+r)^t}$	Present Value Summation